Sommaire

Exercice n°1 *Amérique du Nord J2 - 22 mai 2025*

- $\begin{cases} u_0 = 2 \\ u_{n+1} = \sqrt{u_n} \end{cases}$
- $v_n = \ln(u_n)$

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1. $$\begin{aligned} v_{n+1} &= \ln(u_{n+1}) \\ &= \ln\left(\sqrt{u_n}\right) \\ &= \ln\left({u_n}^{\frac{1}{2}}\right) \\ &= \frac{1}{2}\ln(u_n) \\ v_{n+1} &= \frac{1}{2} \times v_n \end{aligned}$$

Donc (v_n) est géométrique de raison $q = \dfrac{1}{2}$.

2. $v_n = v_0 \times q^n = \ln(u_0) \times q^n = \ln(2) \times \left(\dfrac{1}{2}\right)^n$

3. $v_n = \ln(u_n)$

Donc : $\ln(2) \times \left(\dfrac{1}{2}\right)^n = \ln(u_n)$.

D'où : $$\begin{aligned} \ln(2) &= \frac{\ln(u_n)}{\left(\frac{1}{2}\right)^n} \\ &= \frac{\ln(u_n)}{\left(\frac{1^n}{2^n}\right)} \\ &= \ln(u_n) \times \left(\frac{2^n}{1^n}\right) \\ \ln(2) &= 2^n \times \ln(u_n). \end{aligned}$$

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Exercice n°2 *Amérique du Nord J2 (secours) - 22 mai 2025*

- $\begin{cases} u_0 = 0 \\ u_1 = \dfrac{1}{2} \\ u_{n+2} = u_{n+1} - \dfrac{1}{4}u_n \end{cases}$
- $w_n = u_{n+1} - \dfrac{1}{2}u_n$

1.
$$\begin{aligned} w_{n+1} &= u_{n+2} - \frac{1}{2}u_{n+1} \\ &= \left(u_{n+1} - \frac{1}{4}u_n\right) - \frac{1}{2}u_{n+1} \\ &= \frac{1}{2}u_{n+1} - \frac{1}{4}u_n \\ &= \frac{1}{2}\left(u_{n+1} - \frac{\frac{1}{4}}{\frac{1}{2}}u_n\right) \\ &= \frac{1}{2}\left(u_{n+1} - \frac{1}{4} \times \frac{2}{1}u_n\right) \\ &= \frac{1}{2}\left(u_{n+1} - \frac{1}{2}u_n\right) \\ w_{n+1} &= \frac{1}{2} \times w_n \end{aligned}$$

Donc (w_n) est géométrique de raison $q = \dfrac{1}{2}$.

2. $w_n = w_0 \times q^n = \left(u_1 - \dfrac{1}{2}u_0\right) \times q^n = \dfrac{1}{2} \times \left(\dfrac{1}{2}\right)^n$

3. $w_n = u_{n+1} - \dfrac{1}{2}u_n \iff u_{n+1} = w_n + \dfrac{1}{2}u_n$

Donc :
$$\begin{aligned} u_{n+1} &= \frac{1}{2} \times \left(\frac{1}{2}\right)^n + \frac{1}{2}u_n \\ u_{n+1} &= \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^n + \frac{1}{2}u_n \\ u_{n+1} &= \left(\frac{1}{2}\right)^{n+1} + \frac{1}{2}u_n. \end{aligned}$$

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Exercice n°3 *Asie J2 - 12 juin 2025*

- $\begin{cases} u_0 = 30 \\ u_{n+1} = \dfrac{1}{2}u_n + 10 \end{cases}$
- $v_n = u_n - 20$

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1. $u_1 = 25$
$u_2 = 22{,}5$

2. $$\begin{aligned} v_{n+1} &= u_{n+1} - 20 \\ &= \frac{1}{2}u_n + 10 - 20 \\ &= \frac{1}{2}u_n - 10 \\ &= \frac{1}{2}\left(u_n - \frac{10}{\frac{1}{2}}\right) \\ &= \frac{1}{2}\left(u_n - 10 \times \frac{2}{1}\right) \\ &= \frac{1}{2}(u_n - 20) \\ v_{n+1} &= \frac{1}{2} \times v_n \end{aligned}$$

Donc (v_n) est géométrique de raison $q = \dfrac{1}{2}$.

3. $v_n = v_0 \times q^n = (u_0 - 20) \times q^n = 10 \times \left(\dfrac{1}{2}\right)^n$

4. $v_n = u_n - 20 \iff u_n = v_n + 20$

Donc : $u_n = 10 \times \left(\dfrac{1}{2}\right)^n + 20$.

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Exercice n°4 *Asie J1 - 05 septembre 2025*

- $\begin{cases} p_1 = 1 \\ p_{n+1} = -0{,}4p_n + 0{,}7 \end{cases}$
- $u_n = p_n - 0{,}5$

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1. $$\begin{aligned} u_{n+1} &= p_{n+1} - 0{,}5 \\ &= -0{,}4p_n + 0{,}7 - 0{,}5 \\ &= -0{,}4p_n + 0{,}2 \\ &= -0{,}4\left(p_n + \frac{0{,}2}{-0{,}4}\right) \\ &= -0{,}4(p_n - 0{,}5) \end{aligned}$$
$$u_{n+1} = -0{,}4 \times u_n$$

Donc (u_n) est géométrique de raison $q = -0{,}4$
et de premier terme $u_1 = p_1 - 0{,}5 = 0{,}5$.

2. $u_n = p_n - 0{,}5 \iff p_n = u_n + 0{,}5$

Or $u_n = u_1 \times q^{n-1} = 0{,}5 \times (-0{,}4)^{n-1}$,

donc : $p_n = 0{,}5 \times (-0{,}4)^{n-1} + 0{,}5$.

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Exercice n°5 *Métropole J1 - 09 septembre 2025*

- $\begin{cases} v_0 = 6 \\ v_{n+1} = 3 - \dfrac{2}{v_n} \end{cases}$
- $w_n = \dfrac{v_n - 1}{v_n - 2}$

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1. $$w_{n+1} = \frac{v_{n+1}-1}{v_{n+1}-2} = \frac{3-\dfrac{2}{v_n}-1}{3-\dfrac{2}{v_n}-2} = \frac{2-\dfrac{2}{v_n}}{1-\dfrac{2}{v_n}} = \frac{\dfrac{2v_n-2}{v_n}}{\dfrac{v_n-2}{v_n}} = \frac{2v_n-2}{\cancel{v_n}} \times \frac{\cancel{v_n}}{v_n-2} = \frac{2(v_n-1)}{v_n-2} = 2 \times \frac{v_n-1}{v_n-2}$$

$w_{n+1} = 2 \times w_n$
Donc (w_n) est géométrique de raison $q = 2$
et de premier terme $w_0 = \dfrac{v_0-1}{v_0-2} = \dfrac{5}{4} = 1{,}25$.

2. $$\frac{w_n-1}{1} = \frac{1}{v_n-2} \iff (w_n-1)\times(v_n-2) = 1\times 1 \iff v_n - 2 = \frac{1}{w_n-1} \iff v_n = \frac{1}{w_n-1} + 2$$

Or $w_n = w_0 \times q^n = 1{,}25 \times 2^n$,

donc : $v_n = \dfrac{1}{1{,}25\times 2^n - 1} + 2$.

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Exercice n°6 *Métropole J2 - 10 septembre 2025*

- $\begin{cases} p_0 = 0 \\ p_{n+1} = 0{,}2p_n + 0{,}3 \end{cases}$
- $u_n = p_n - \dfrac{3}{8}$

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1. $$\begin{aligned} u_{n+1} &= p_{n+1} - \frac{3}{8} \\ &= 0{,}2p_n + 0{,}3 - \frac{3}{8} \\ &= 0{,}2p_n - 0{,}075 \\ &= 0{,}2\left(p_n - \frac{0{,}075}{0{,}2}\right) \\ &= 0{,}2\left(p_n - \frac{3}{8}\right) \end{aligned}$$

$u_{n+1} = 0{,}2 \times u_n$

Donc (u_n) est géométrique de raison $q = 0{,}2$
et de premier terme $u_0 = p_0 - \dfrac{3}{8} = -\dfrac{3}{8}$.

2. $u_n = p_n - \dfrac{3}{8} \iff p_n = u_n + \dfrac{3}{8}$

Or $u_n = u_0 \times q^n = -\dfrac{3}{8} \times 0{,}2^n$,

donc : $$\begin{aligned} p_n &= -\frac{3}{8} \times 0{,}2^n + \frac{3}{8} \\ p_n &= \frac{3}{8}\left(-0{,}2^n + 1\right) \end{aligned}$$

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Exercice n°7 *Amérique du Sud J2 - 14 novembre 2025*

- $\begin{cases} v_0 = \ln(4) \\ v_{n+1} = \ln\left(-1+2\mathrm{e}^{v_n}\right) \end{cases}$
- $w_n = -1 + \mathrm{e}^{v_n}$

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1. $w_{n+1} = -1 + \mathrm{e}^{v_{n+1}}$

$$\begin{aligned} &= -1 + \mathrm{e}^{\ln\left(-1+2\mathrm{e}^{v_n}\right)} \\ &= -1 + \left(-1 + 2\mathrm{e}^{v_n}\right) \\ &= -2 + 2\mathrm{e}^{v_n} \\ &= 2\left(-1 + \mathrm{e}^{v_n}\right) \end{aligned}$$

$w_{n+1} = 2 \times w_n$

Donc (w_n) est géométrique de raison $q = 2$.

2. $w_n = -1 + \mathrm{e}^{v_n} \iff \mathrm{e}^{v_n} = w_n + 1 \iff v_n = \ln(w_n + 1)$

Or $w_n = w_0 \times q^n = (-1 + \mathrm{e}^{v_0}) \times q^n = 3 \times 2^n$,

donc : $v_n = \ln(3 \times 2^n + 1)$.

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